Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Sound and Vibration 290 (2006) 410–424
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Reducing the amplitude of vibration at resonances by phase modulation

S.-M. Wang*, Q.-S. Lu, Q. Wang, P. Xu

Science School, Beijing University of Aeronautics and Astronautics, Beijing 100083, PR China

Received 20 January 2004; received in revised form 26 August 2004; accepted 2 April 2005
Available online 21 June 2005

Abstract

The mechanism of phase modulation method for reducing the vibration amplitude of vibrating systems accelerating or decelerating through its resonance is presented. The method is theoretically used to rotor and one degree of freedom oscillator models with numerical simulation; numerical results show that the maximum amplitude can be reduced by about 15–20% with phase modulation in comparison with the constantly accelerating case. Possible implementation of the method was introduced. Experiments with a cantilever sheet subjected pulses excitation were conducted to confirm the method; experimental results show that the lateral vibration amplitude of the cantilever at the passage of its first resonance can be reduced about 18% with phase modulation method.

1. Introduction

During run-up or run-down, machines are subject to oscillating forces of increasing or decreasing frequency. If resonances have to be passed, large vibration amplitude at resonance is one of the most serious concerns. Especially for rotating machines, large lateral vibration amplitude may cause tip-rubs of rotor and guard as well as increased bearing reactions [1–3], and even some of the situations can be catastrophic. In order to avoid the damage due to rubbing, nominal tip clearances are specified well in excess of those necessities for steady-state supercritical

*Corresponding author.
E-mail address: wsmbj@yahoo.com.cn (S.-M. Wang).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2005.04.009

Nomenclature

M	mass of the oscillator (kg)
C	damping constant (N s/m)
k_1, k_2	stiffness constants (N/m)
l_1, l_2	un-stretched length of springs (m)
m	mass of the rotor (kg)
e	eccentric distance of the rotor (m)
P	potential energy of springs (J)
$\mathbf{F}$	vector of excitement force (N)
x, y	vibration displacements (m)
$\bar{x}$ $\bar{y}$	dimensionless vibration displacements
φ	phase of the rotor (rad)
$\varphi(\tau)$	dimensionless phase of the rotor or oscillating force
θ	angle between normal and resultant inertia force (rad)
$\theta(\tau)$	dimensionless angle between normal and resultant inertia force
δ	dimensionless damping
ω_n	the nth natural frequency (1/s)
$\bar{P}$	dimensionless potential of the springs
F	magnitude of dimensionless excitement force
κ	ratio of the masses
τ	dimensionless time
ψ	phase of the excitement force (rad)
$\psi(\tau)$	dimensionless phase of the excitement force
γ_1, γ_2	dimensionless lengths of un-stretched springs
η	ratio of stiffness
T	dimensionless kinetic energy of the oscillator
ρ	phase of dimensionless vibration velocity
V	dimensionless total vibration energy
β^*	phase difference between the vibration velocity and excitement force
A_c, A_d	dimensionless rate of increase/decrease the excitement frequency
ε	constant
f_r, f_a	retreating and advancing point
a_m	maximum amplitude at resonance

operation. This degrades the turbo-machine's aerodynamic performance. In particular, large clearances create low efficiencies for both turbines and compressors.

In the case of non-stationary process there is, in general, an interaction between the oscillating system and its driving mechanism. While approaching a resonance, the driving mechanism pumps energy into the oscillation, namely, the oscillating forces do positive work; whereas after the passage of the resonance, a part of the vibration energy flows back into the driving mechanism or the oscillating forces do negative work. These imply that the energy transmission between the oscillating system and driving mechanism depends on the phase relations between the response and excitation. Since the non-stationary response cannot be analytically expressed even for linear oscillator subjected to oscillating force with linear increased frequency. Therefore, the effects of phases on the vibration energy or amplitude are not obvious and may also be neglected. Markert and Seidler [4] gave an exact analytical estimate of the maximum amplitude and resonant frequency as well as the phase at resonance. The history of amplitude and phase is also important in understanding the mechanism of the vibration of a system passing through resonance and finding the methods to reduce the vibration amplitude. In industry practices, the general method to avoid large amplitude resonance at critical speeds is to take as large acceleration as possible to drive the system passing through the critical speed [5]. Indeed this measure is to ensure the phases of oscillating system and driving mechanism keeping as fewer synchronic cycles as possible. It takes the advantage of phase relations. On the other hand, this method may be limited by the allowable maximum acceleration that depends on power of driving mechanism. Consequently, the following question was raised: for limited acceleration or rate of increasing frequency, can the maximum vibration amplitude be further reduced? Millsaps and Reed [6] at first suggested an acceleration scheduling method based on the amplitude–frequency characteristics of different accelerations: firstly took a larger acceleration to accelerate the rotor to an adequate state, then changed the acceleration to a smaller one to keep on accelerating the rotor, and expected to reduce the maximum amplitude. Although they did not pay attention to the phase–frequency characteristics: the amplitude will be increased in any case if switch accelerations at the phase of amplitude increase. However, they established the way of reducing the vibration amplitude by alternating acceleration. Based on their works, Wang et al. [7] proposed a way of modulating the phase: let the excitement frequency go through an increase–decrease–increase procedure to modulate the phase relations of excitement and response and to reduce the vibration amplitude. In other words, adjust the history of phases to reduce the energy pumped to oscillating system. In this work, the mechanism of phase modulation was explained and the method was generalized to common vibrating system other than rotor system. An experimental confirmation was conducted. In the following section, the mechanism of the phase modulation method was introduced. In Section 3, the relations between the vibration amplitude and the phase difference of excitation and response was analyzed. In Section 4, an empirical way of implementing the method was presented. In Section 5, phase modulations of run-up and run-down cases were shown with numerical simulation. In Section 6, an experimental confirmation was implemented.

2. The effect of the excitement phase on the vibration amplitude

For constant excitement frequency, both the vibration amplitude and the phase difference between excitation and response are constant; while they are changed with varied excitement frequencies; and the increase/decrease of vibration amplitude closely relates to the phase difference and/or the history of excitement frequency.

2.1. Numerical results

We use the following model to numerically show the relations between vibration amplitude and phase difference.

The two degrees of freedom damped spring-mass oscillator in Fig. 1 has mass M and damping constant c; its two springs have stiffness constants k_1 and k_2, as well as un-stretched lengths l_1 and l_2, respectively. The oscillator is subjected to an inertial force caused by a rotor with mass m and eccentric distance e, which rotates about point O with phase φ.

Its equations of motion are

$$\begin{aligned}(M+m)\ddot{x}+c\dot{x}+\frac{\partial P}{\partial x}&=em(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi),\\ (M+m)\ddot{y}+c\dot{y}+\frac{\partial P}{\partial y}&=em(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi),\end{aligned} \quad (1)$$

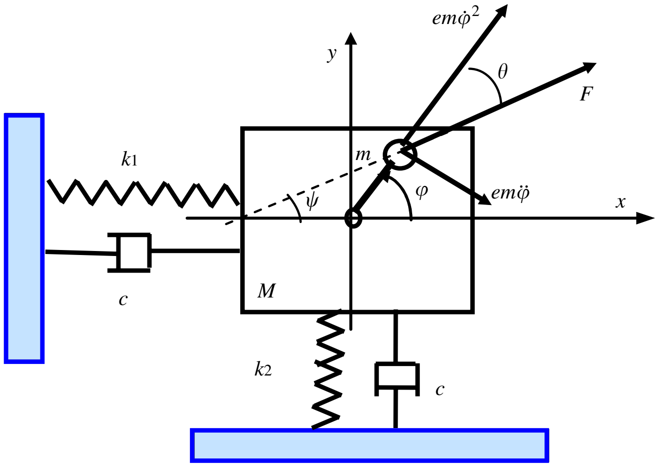

Fig. 1. Two degrees of freedom vibration system model with rotary component.

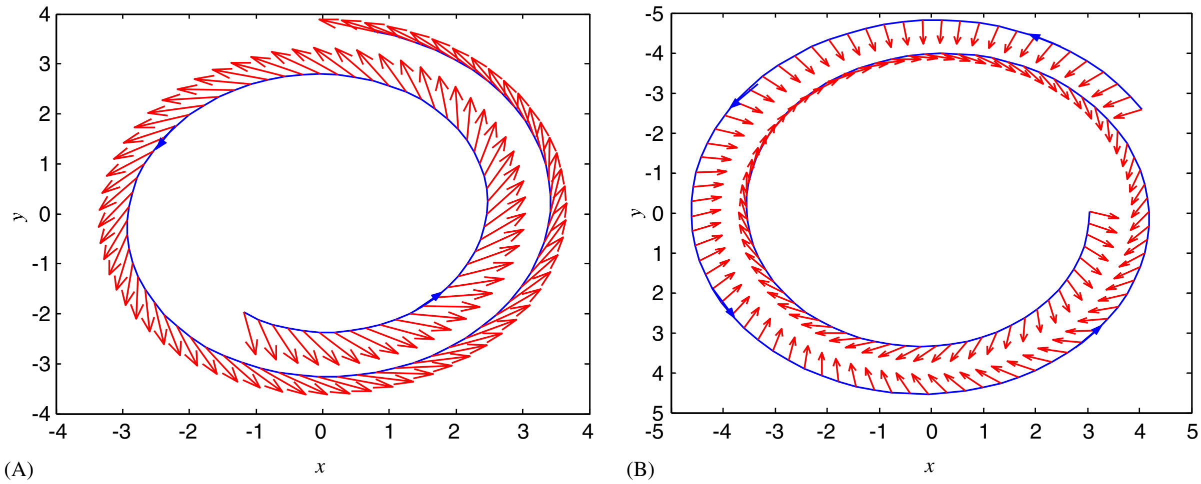

Fig. 2. Phases of excitement force and vibration velocity for (A) increased amplitude and (B) decreased amplitude.

where the dot denotes differentiation with respect to time t, P is the potential energy of springs

$$P = \frac{k_1}{2}\left[x^2 + 2l_1 x + y^2 - 2l_1\sqrt{(x+l_1)^2 + y^2}\right] + \frac{k_2}{2}\left[x^2 + 2l_2 y + x^2 - 2l_2\sqrt{(y+l_2)^2 + x^2}\right].$$

To show the relations between the phase difference and the vibration amplitude, Eq. (1) is numerically integrated with constant angular acceleration $\ddot{\varphi}(t)$. Two segments of displacement curves corresponding to increased and decreased amplitudes are shown in Fig. 2(A) and (B), respectively, where arrows represent the direction of inertial force.

In Fig. 2, continuous curves represent vibration displacements, arrows started from curve show the direction of the excitement force **F** at that moment. Here, **F** is the resultant inertia force caused by the rotor. Fig. 2(A) shows increased vibration amplitude, where the phase angle between the vibration velocity **v** (the tangent direction of the displacement curve) and the force is less than $\pi/2$; therefore, one has $\mathbf{F}\cdot \mathrm{d}\mathbf{r} = \mathbf{F}\cdot\mathbf{v}\,\mathrm{d}t > 0$, or the driving mechanism pumps energy into the oscillation. Since the damper always dissipates energy, the vibration energy or amplitude can be increased only at such phase. When the angle is greater than $\pi/2$ shown as Fig. 2(B), the vibration amplitude will definitely decrease, at such phase the vibration energy flows back into the driving mechanism except the energy dissipated by damping. Here, we call this phase as 'amplitude increased/decreased phase', accordingly it is lesser/greater than $\pi/2$, respectively.

2.2. Analytical result

To get analytical results, using θ, the angle between normal and resultant inertia force, we introduced the following transformations:

$$\frac{\ddot{\varphi}}{\sqrt{\ddot{\varphi}^2+\dot{\varphi}^4}} = \sin\theta, \quad \frac{\dot{\varphi}^2}{\sqrt{\ddot{\varphi}^2+\dot{\varphi}^4}} = \cos\theta. \tag{2}$$

By means of above transformations Eq. (1) can be rewritten as

$$\begin{aligned}(M+m)\ddot{x}+c\dot{x}+\frac{\partial P}{\partial x} &= em\sqrt{\dot{\varphi}^4+\ddot{\varphi}^2}\cos(\varphi-\theta),\\ (M+m)\ddot{y}+c\dot{y}+\frac{\partial P}{\partial y} &= em\sqrt{\dot{\varphi}^4+\ddot{\varphi}^2}\sin(\varphi-\theta),\end{aligned} \tag{3}$$

where $\varphi-\theta=\psi$ represents the phase angle of the excitement (inertia) force shown as Fig. 1.

The dimensionless form of Eq. (3) is

$$\begin{aligned}\bar{x}''+2\delta\bar{x}'+\frac{\partial\bar{P}}{\partial\bar{x}} &= F\cos\psi(\tau),\\ \bar{y}''+2\delta\bar{y}'+\frac{\partial\bar{P}}{\partial\bar{y}} &= F\sin\psi(\tau),\end{aligned} \tag{4}$$

where $\bar{x}=x/e$, $\bar{y}=y/e$, $2\delta=c/(M+m)\omega_1$, $\omega_1^2=k_1/(M+m)$, $\bar{P}=P/k_1e^2$, $F=\kappa\sqrt{\varphi''^2+\varphi'^4}$, $\kappa=m(m+M)$, $\psi(\tau)=\varphi(\tau)-\theta(\tau)$ and $\tau=\omega_1 t$. The prime denotes differentiation with respect to the non-dimensional time τ.

The dimensionless potential is

$$\begin{aligned}\bar{P} &= \frac{P}{k_1e^2}\\ &= \frac{1}{2}\left[\bar{x}^2+2\gamma_1\bar{x}+\bar{y}^2-2\gamma_1\sqrt{(\bar{x}+\gamma_1)^2+\bar{y}^2}\right]\\ &\quad+\frac{\eta}{2}\left[\bar{x}^2+2\gamma_2\bar{y}+\bar{x}^2-2\gamma_2\sqrt{(\bar{y}+\gamma_2)^2+\bar{x}^2}\right],\end{aligned}$$

where $l_1/e=\gamma_1$, $l_2/e=\gamma_2$ and $k_2/k_1=\eta$.

The vibration energy of the rotor is

$$V = \bar{P} + \frac{1}{2}(\bar{x}'^2 + \bar{y}'^2)$$

and its derivative with respect to τ is

$$V' = \left(\frac{\partial \bar{P}}{\partial \bar{x}} + \bar{x}''\right)\bar{x}' + \left(\frac{\partial \bar{P}}{\partial \bar{y}} + \bar{y}''\right)\bar{y}. \tag{5}$$

Using Eq. (4), Eq. (5) can be written as

$$V' = \bar{x}'(F\cos\psi(\tau) - 2\delta\bar{x}') + \bar{y}'(F\sin\psi(\tau) - 2\delta\bar{y}'). \tag{6}$$

By virtue of the dimensionless kinetic energy $T = \frac{1}{2}(x'^2 + y'^2)$, we express vibration velocities as

$$\bar{x}' = \sqrt{2T}\cos\rho, \quad \bar{y}' = \sqrt{2T}\sin\rho, \tag{7}$$

where ρ gives the phase of vibration velocity and substitution of Eq. (7) into Eq. (6) yields

$$\bar{V}' = \sqrt{2T}\left[F\cos(\rho - \psi(\tau)) - 2\delta\sqrt{2T}\right], \tag{8}$$

where $\rho - \psi(\tau) = \beta^*$ expresses the phase difference between the vibration velocity (response) and the excitement force (excitation). By Eq. (8) it can be concluded that the increase/decrease of vibration energy depends on phase difference β^*: the vibration energy can be increased only if $\cos(\rho - \psi(\tau)) > 0$, namely, the phase difference satisfies $-\pi/2 < \beta^* < \pi/2$ (or modulo 2π), otherwise the vibration energy will be decreased. This conclusion coincides with the numerical results in Section 2.

3. The mechanism of phase modulation

For non-stationary excitation, vibration energy or amplitude depends on the phase difference between excitement and response. And the change of phase difference follows the varied excitement frequency. This makes the basis of phase modulation method.

3.1. Phase history for constantly increased excitement frequency

In the case of monotonically increased frequency, the vibration amplitude is increased before the passage of the resonance, or the phase difference remains in ‘amplitude increased phase’. During this period, the phase difference goes from $\pi/2$ to $-\pi/2$ and approaches $-\pi/2$ near resonant frequency. After the passage of resonance, the phase difference further decreases and transits into $[-3\pi/2, -\pi/2]$ via $-\pi/2$. Along with the increased excitement frequency the phase difference alternates between ‘amplitude increased phase’ and ‘amplitude decreased phase’; and so does the amplitude.

3.2. Phase change for ‘to and fro’ excitement frequency

The phase modulation is carried out before the passage of resonance. To modulate the phase difference, the excitement frequency is not monotonically increased, but decreased at an adequate magnitude called 'retreating point'. Consequently, the phase difference will stop decreasing toward $-\pi/2$, but increases toward $\pi/2$ and finally transits to $[\pi/2, 3\pi/2]$ via $\pi/2$, which is an 'amplitude decreased phase' in the sense of modulo 2π. The excitement frequency if kept decreased, the phase difference will also alternate between 'amplitude decreased phase' and 'amplitude increased phase'; when it enters the first amplitude decreased one we say the phase has been modulated. After the phase is modulated, the excitement frequency is increased again at an adequate moment called 'advance point'. At this time, the phase will remain in the amplitude decreased one for a while and then transit to amplitude increased one. Correspondingly, the vibration amplitude will firstly decrease and then increase via its minimum. If the phase is adequately modulated, the minimum amplitude will be smaller than the amplitude of un-modulated case at the same frequency. Following the increased frequency, the vibration amplitude approaches its maximum at resonance. In summary, the vibration amplitude goes an increase–decrease procedure during phase modulation; and when the excitement frequency is increased again, the amplitude still keeps decreasing until it approaches to its minimum. After the excitement frequency completes a decrease–increase cycle the vibration amplitude has been reduced. And the amplitude at resonance correspondingly is reduced.

3.3. Example 1

Here, we numerically demonstrate the process of phase modulation by integrating Eq. (4). For the following parameters and initial conditions:

$$\delta = 0.01, \quad \eta = 1, \quad \frac{m}{m+M} = 0.15,$$
$$x|_{\tau=0} = 0, \quad x'|_{\tau=0} = 0, \quad y\big|_{\tau=0} = 0, \quad y'\big|_{\tau=0} = 0, \quad \varphi(\tau)\big|_{\tau=0} = 0, \quad \varphi'(\tau)\big|_{\tau=0} = 0.$$

At first the angular velocity of the rotor $\varphi'(\tau)$ is increased from 0 to 0.96 with acceleration $\varphi''(\tau) = 0.01$, and then it is decreased to 0.89 with rate $\varphi''(\tau) = 0.001$. Finally, it is increased again with acceleration $\varphi''(\tau) = 0.01$ to pass through the resonance. The numerical results of above procedures are shown in Fig. 3.

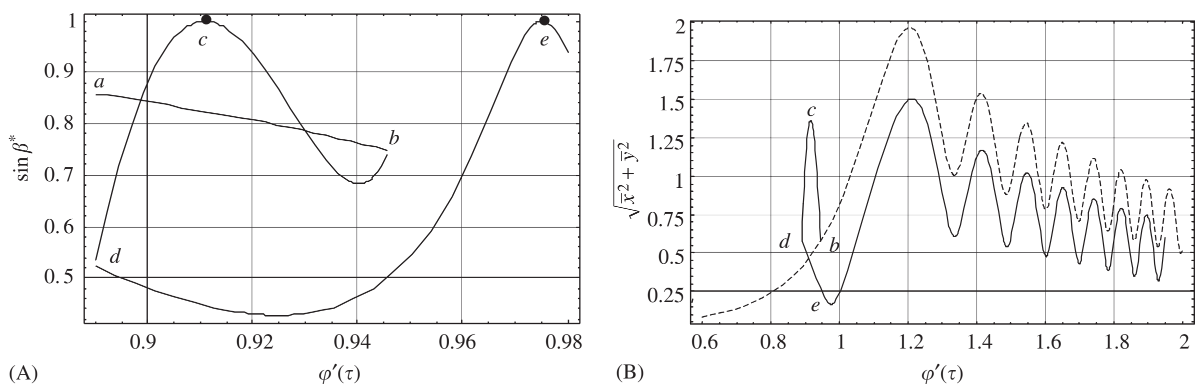

Fig. 3. (A) Sine of the phase difference β^* and (B) vibration amplitude for the 'to and fro' frequency.

In Fig. 3(A), the vertical axis gives the sine of the phase difference, and the horizontal axis is the angular velocity of the rotor $\varphi'(\tau)$. Curve segment *ab* corresponds to the angular velocity being increased from 0.89 to 0.96 and *bcd* shows the phase modulation process, that is, the angular velocity is decreased from 0.96 to 0.89. Along with decreased angular velocity, the phase difference changes from 0.86 (at $\varphi'(\tau) = 0.96$) to 2.58 (at $\varphi'(\tau) = 0.89$), via $\pi/2$; while it is 1.03 when the angular velocity is increased to the same magnitude $\varphi'(\tau) = 0.89$.

Fig. 3(B) shows the corresponding vibration amplitude. At the beginning of phase modulation, the vibration amplitude, shown as *bc*, keeps increasing until it approaches its peak at the phase of about $\pi/2$, and then begins to decrease. When the angular velocity is increased again, the amplitude decreases for a while shown as *de* and then increases. Comparing to the case of monotonically increased angular velocity shown as dashed line, the maximum amplitude is reduced.

3.4. Example 2

This method can be generalized to vibration system subjected to various forcing functions other than inertia force, the following example is a damped spring-mass oscillator subjected to pulse excitement for passing through its resonance

$$\bar{x}'' + 2\delta\bar{x}' + \bar{x} = F(0.5 + 0.5\,\mathrm{sign}[\cos\psi(\tau)]), \tag{9}$$

where sign is sign function: it takes -1 if $\cos\psi(\tau) < 0$, otherwise takes 1. Let $\delta = 0.0024$, $\psi''(\tau) = 0.0055$ and $F = 0.2 \times 6^2/(\bar{x} - 6)^2$. To modulate the phase, when the pulse rate, namely, $\psi'(\tau)$ is increased from 0 to 0.93 ($\tau = 169$), it is decreased to 0.88 ($\tau = 225$) with constant rate $\psi''(\tau) = -0.00092$; and then it is increased again with rate $\psi''(\tau) = 0.0055$ to pass through the resonance. Fig. 4(A) shows the vibration displacement with constantly increasing pulse-rate and (B) the vibration displacement in phase modulation case.

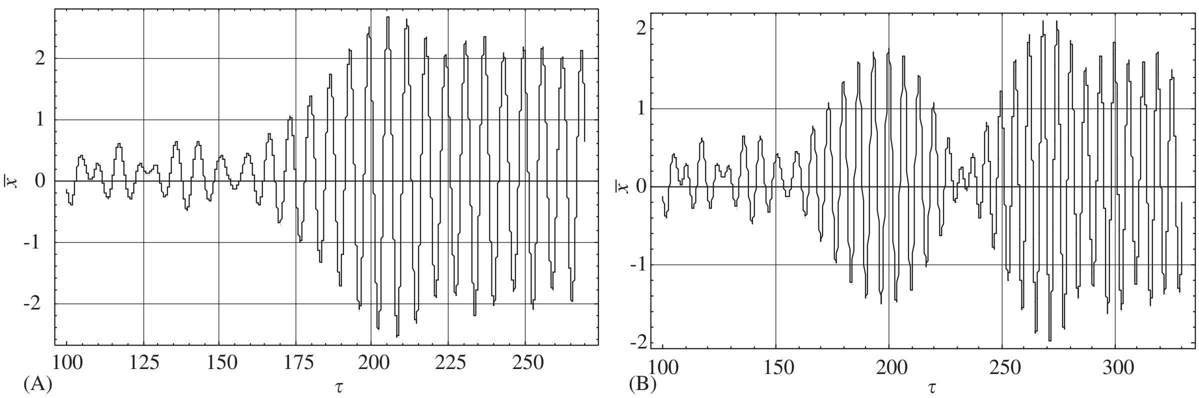

Fig. 4. Amplitudes of an oscillator subjected to pulses excitation for pass through its resonance: (A) constantly increased pulse rate and (B) phase modulation case.

Remarks.

1. For one degree of freedom oscillator, the vibrating displacement and velocity can be formally expressed as $\bar{x} = \sqrt{2V}\cos\vartheta$ and $\bar{x}' = \sqrt{2V}\sin\vartheta$, where $V = \frac{1}{2}(\bar{x}'^2 + \bar{x}^2)$ is the vibration energy. Therefore, the vibration amplitude can be taken as $\sqrt{2V}$.
2. At the moment of alternating acceleration, the continuity of variables ($\bar{x}$, $\bar{x}'$, ψ and ψ') should be kept, that is, the end states of last procedure are taken as the initial conditions of the next one.

4. An empirical way for phase modulation

The essentials of phase modulation are to choose 'retreating point', 'advancing point' and the decrease rate of the excitement frequency. For a constant increase rate of excitement frequency $A_c = \psi''(\tau)$, the strategies for determining these parameters are discussed.

4.1. An empirical way of phase modulation

The above example and discussion show that the phase difference can be modulated by decreasing the excitement frequency for a period. Here, trial and error was used for finding adequate decrease rate, 'retreating point' and 'advancing point'.

4.1.1. The decrease rate of frequency for phase modulation A_d

To determine the decrease rate, the first issue is the peak vibration amplitude during the procedure of phase modulation. It should be controlled not to exceed the amplitude at resonance. This peak depends on both the decrease rate and the 'retreating point': for a given retreating point, the larger the decrease rate, the smaller the peak is; while for a given decrease rate, the peak will get bigger and bigger along with the 'retreating point' approaching to resonant frequency. To ensure this peak is smaller than amplitude at resonance, adequate decrease rate and 'retreating point' are required. Another consideration is to prolong the process of amplitude being reduced but not to decrease the excitement frequency too much during phase modulation. This requires a relatively small decrease rate.

The vibration amplitude at resonance depends on the increasing rate of excitement frequency: the larger the increasing rate, the smaller the maximum amplitude is. Taking into account these factors, we suggested that the decrease rate is proportional to the increase rate of the excitement frequency A_c

$$A_d = -\varepsilon A_c, \tag{10}$$

where ε is a constant between 0.1 and 0.2.

4.1.2. Retreating point F_r

Once the decrease rate is determined, retreating point can be chosen by trial and error in the following procedure:

1. Finding the resonant amplitude a_m in the case of constantly increased excitement frequency A_c.
2. Choosing a retreating point, once the excitement frequency is increased to this point with the rate A_c, immediately decrease the excitement frequency with the rate determined by Eq. (10).

3. Finding the peak amplitude of above procedure and comparing it to the maximum amplitude a_m, if the former is less than the latter about 20%, then this 'retreating point' is taken as the expected one. Otherwise, adjust the retreating point and repeat procedure 2 and 3 until adequate retreating point in found.

For the increasing rate A_c ranged from 0.005 to 0.02, and damping coefficient δ ranged from 0.001 to 0.02 we suggest the following analytical approximation for 'retreating point':

$$f_r = 0.98\,\omega_n - 3\,A_c - 0.00005/A_c + 0.5\delta. \tag{11}$$

It was obtained by means of linear fitting, where ω_n is the nth natural frequencies of vibration system.

4.1.3. The advancing point F_a

During the phase modulation, namely, the process of decreasing excitement frequency, the vibration amplitude will first increases and then decrease. The 'advancing point', at where the excitement frequency is increased again, locates at the decrease period of amplitude. After testing different 'advancing points', one of them will be chosen as 'advancing point', if it produces the smallest resonant amplitude when the excitement frequency is increased again. Here, we gave an approximate way of determining the advancing point as

$$f_a = f_r/1.028 - 2.6A_c + 0.000008/\delta \tag{12}$$

4.2. Phase modulation for run-down case

In the case of run-down, there is also peak amplitude at the passage of resonance. The maximum amplitude can be reduced by phase modulation in the same mechanism as run-up case. The process is similar to the run-up case but in the symmetric way. The 'retreating' and 'advancing' points are, respectively, given by

$$f_r = 1.04\omega_n + 3A_c + 0.00005/A_c - 0.5\delta, \tag{13}$$

$$f_a = 1.01 f_r + 2.6A_c + 0.000008/\delta. \tag{14}$$

4.3. Possible implementation of the method in practice

For real implementation of this method, equipments for testing vibration displacements and controlling the excitement frequency are required. Vibration displacements should be directly or indirectly expressed as the function of excitement frequency, namely, an amplitude–frequency history. In practice, we suggest the following procedures:

1. Determining the allowable increase rate of excitement frequency A_c according to driven system; and testing the natural frequency of the oscillating equipment.
2. Using the rate A_c to run-up the equipment passing through its resonance, measuring the history of vibration displacement and finding the maximum vibration amplitude. Usually, the resonance takes place at a frequency that is greater than the natural one [4].

3. Choosing the decrease rate according to Eq. (10) , and taking a preliminary retreating point between $0.9\omega_n$ and $0.95\omega_n$; if large amplitude is harmful to the equipment, a smaller retreating point and a larger decrease rate should be taken for safety consideration.
4. Running down oscillating system to static with chosen decrease rate after running up to retreating point, testing vibration displacement, getting amplitude–frequency history and finding the maximum vibration amplitude during this process.
5. Comparing the two maximum vibration amplitudes recorded from steps 2 and 4; if the latter is smaller than the former about 10–25%, then the decrease rate and retreating point are adequate, otherwise adjust the retreating point and repeat step 4.
6. Making the envelop curve of the displacement history measured in step 4, and drawing a horizontal line passing through the starting point of the envelop curve, an advancing point is then taken as the intersection point of the line and envelop curve.

5. Numerical examples of phase modulation

5.1. Run-up case of two degrees of freedom with split resonance

For vibration system described by Eq. (4) with parameters $\delta = 0.01$, $\eta = 1.44$, $\gamma_1 = \gamma_2 = 50$, $\kappa = 0.15$ The system has split natural frequencies 1 and 1.2. For passing through the first resonance, the frequency is decreased to $f_a = 0.899$ with rate $A_d = -0.001$ after it is increased to $f_r = 0.95$ with increasing rate $A_c = \varphi''(\tau) = 0.01$. And then it is increased again. When the frequency approaches to the second retreating point $f_{r2} = 1.146$, it is decreased to second advancing point $f_{a2} = 1.0896$ with rate $A_d = -0.001$; and then is increased to work frequency. The numerical results were shown in Fig. 5.

Remarks.

3. If two resonant frequencies are at a distance from each other, it may happen that the modulated maximum amplitude is greater than the first peak amplitude without phase modulation, since the second resonance amplitude is usually the bigger one. Therefore, only the phase modulation is necessary before the passage of second resonance.
4. If the two resonant frequencies are close to each other, there is no space for second phase modulation; however, the maximum amplitude can still be reduced by the first phase modulation.

5.2. Run-down case of two degrees of freedom with split resonance

Take the same parameters as a run-up case, suppose the excitement frequency will run-down from 1.8 to 0 with rate $A_c = \varphi''(\tau) = -0.01$. The passages of two resonances will also result in amplitude peaks. To modulate the phase, the frequency is increased to 'advancing point' 1.317 with rate $A_d = 0.001$ after it is decreased to 'retreating point' 1.278; and then it is decreased again. These parameters are calculated by expressions (13) and (14), respectively. Here, we did not modulate the phase before passing through the second resonance, since this peak is less than the maximum amplitude after first phase modulation. The numerical results were shown in Fig. 6.

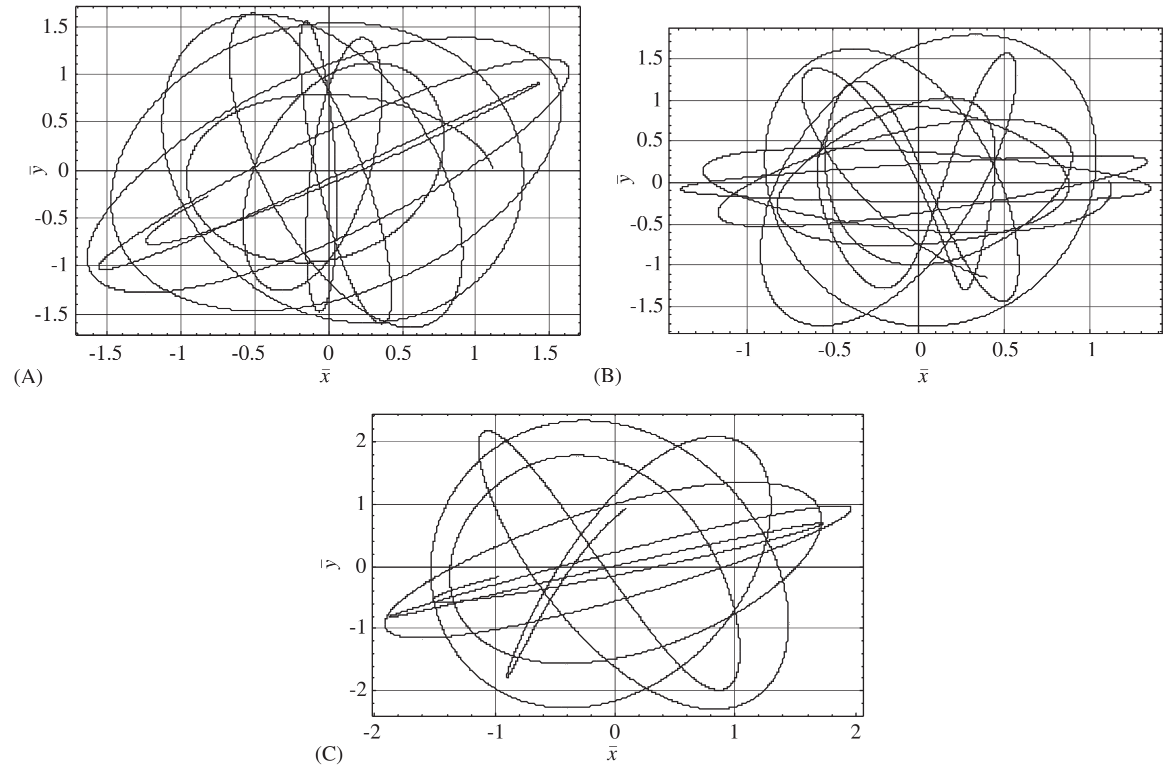

Fig. 5. Vibration amplitudes during (A) second phase modulation, $\varphi'(\tau)$ decreasing from 1.146 to 1.0896, (B) $\varphi'(\tau)$ increasing from 1.0896 to 1.4 of phase modulation case and (C) $\varphi'(\tau)$ increasing from 1.0 to 1.4 without phase modulation.

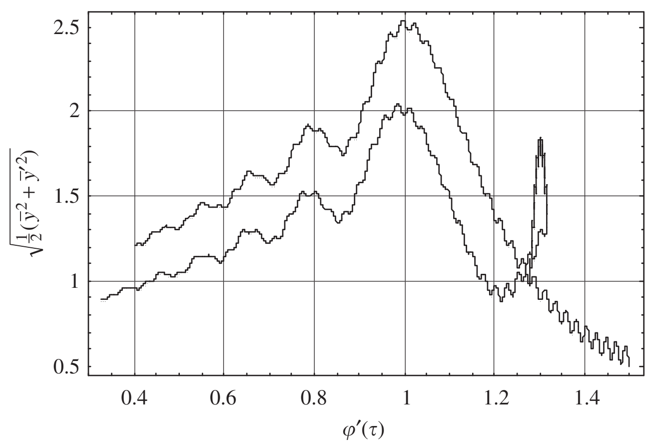

Fig. 6. The vibration energy in y-direction of run-down case: dotted line shows constantly decreased frequency case; solid line represents phase modulation case.

6. Experimental confirmation

6.1. Overview

Experiments were conducted to verify that phase modulation is capable of reducing amplitude at resonance in real hardware. We chose cantilever subjected pulses excitement to pass through the resonance; it is relatively simple in implementing the experiments. The numerical result of the model is given by the second example of Section 3.

6.2. Facility description

The experimental facility is based on a cantilever steel sheet clamped on a bench clamp, which has a dimension $267 \times 23 \times 2\,\text{mm}^3$ shown as Fig. 7. A button magnet was stuck on the free end of the sheet. A magnetic coil was used to exert magnetic force on the button magnet. Two strain gauges connected in half-bridge were stuck on the clamped end for testing the flexure strain of the sheet. The output voltage in Wheatstone bridge was recorded by computer through a USB A/C converter. The excitement pulses signal with varied frequency was generated and sent out through a USB port to the control end of a solid-state relay that supplies driven current with 15 V for the magnet coil. The natural frequency and damping coefficient of the sheet were tested by the attenuation history of the free vibration; which are 13.2 and 0.2011, respectively.

6.3. Results

The exciting pulses were generated by sign function

$$I = \operatorname{sign}\left(\cos\left[2\pi\left(\frac{1}{2}at^2 + f_0 t + \varphi_0\right)\right]\right).$$

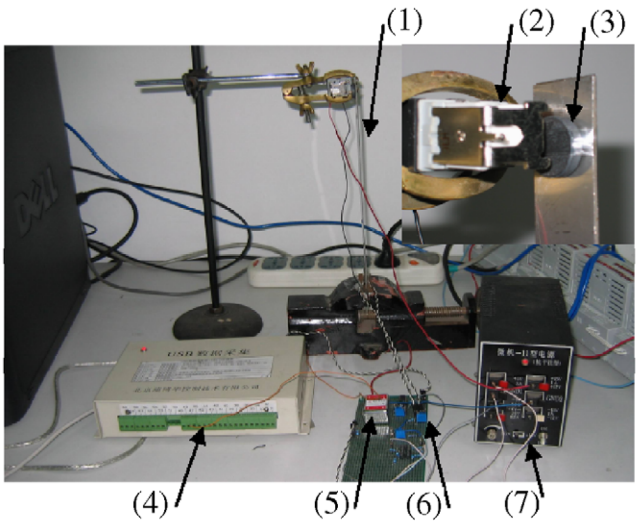

Fig. 7. Experimental facility (1) steel sheet, (2) magnetic coil, (3) button magnet, (4) USB A/D converter, (5) solid state relay, (6) amplifier circuit and (7) DC powers.

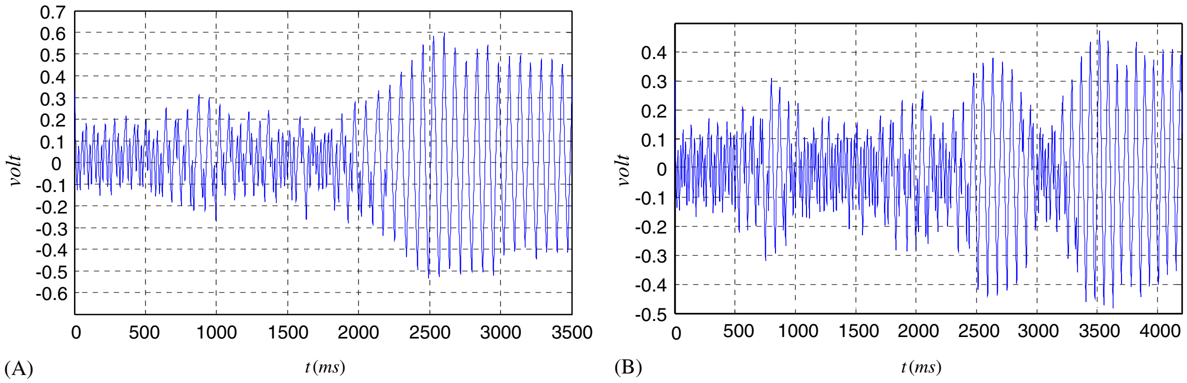

Fig. 8. Voltage signal of strain gauges: (A) Constant pulse rate case and (B) phase modulation case.

The comparison case was produced with parameters $a = 6, f_0 = 0$, $\varphi_0 = 0$ and time t for 3 s to pass through the resonance. The acceleration $a = 6$ is equivalent to dimensionless acceleration $\bar{a} = 0.0055$. The measured strain was shown in Fig. 8(A).

For phase modulation, the excitement frequency was firstly increased from 0 to 12.3 by taking $a = 6, f_0 = 0$, $\varphi_0 = 0$ and running time t for 2.05 s; then it is decreased to 11.6 with taking $a = -1$, $f_0 = 12.3$, $\varphi_0 = 12.6075$ and running time t for 0.7 s. Finally, we took $a = 6$, $f_0 = 11.6$ and $\varphi_0 = 20.9725$, and run time t for 1 s to pass through resonance. The measured strain signal was shown in Fig. 8(B). The maximum displacement is reduced by about 18%.

The first mode vibration was modeled by Eq. (9) in Section 3.4, the magnitude of magnetic force exerted on the button magnet was approximated by $F = 0.2 \times 6^2/(\bar{x} - 6)^2$.

7. Conclusions

The validated cantilever model was used to show that it is possible to reduce lateral vibration at resonance by phase modulation method. Preliminary experimental results support this conclusion.

While all the efforts described focused on simple models, the general technique should be applicable to real vibrating systems only if these systems have the same phase–frequency characteristics as that of systems described by Eq. (1) or Eq. (9). It should be possible to further reduce the vibration amplitude and/or the total vibration energy by developing optimum decrease rate, retreating point and advancing point for a given application. However, such an optimum phase modulation may require high accurate control of the excitement frequency. Also, rapid changes in the acceleration will excite unexpected vibration. However, if this technique can be implemented in real machinery, it will be efficient since it does not require any auxiliary equipments but a programmed run-up procedure.

Acknowledgment

This work was partially supported by the National Natural Science Foundation of China under Grant No. 10272008, and 10172011.

References

[1] F.K. Choy, J. Padovan, Nonlinear transient analysis of rotor-casing rub events, *Journal of Sound and Vibration* 113 (1987) 529–545.

[2] G.X. Li, M.P. Paidoussis, Impact phenomena of rotor-casing dynamical systems, *Nonlinear Dynamics* 5 (1994) 53–70.

[3] S. Yanabe, S. Kanneko, N. Shibata, Rotor vibration due to collision with annular guard during passage through critical speed, *Journal of Vibration and Acoustics* 120 (1998) 549–550.

[4] R. Markert, M. Seidler, Analytically based estimation of the maximum amplitude during passage through resonance, *International Journal of Solids and Structures* 38 (2001) 1975–1992.

[5] R.L. Fearn, K.T. Millsaps, Constant acceleration of an undamped simple vibrator vibrator through resonance, *Journal of the Royal Aeronautical Society* (1967) 567–573.

[6] K.T. Millsaps, G.L. Reed, Reducing lateral vibrations of a rotor passing through critical speeds by acceleration scheduling, *Journal of Engineering for Gas Turbines and Power* 120 (1998) 615–620.

[7] S.-M. Wang, Q.-S. Lu, E.H. Twizell, Reducing lateral vibrations of a rotor passing through critical speeds by phase modulating, *Journal of Engineering for Gas Turbines and Power* 125 (2003) 766–771.